The Differential Amplifier

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BJT Differential Pair

• Differential pair circuits are one of the most widely used circuit building blocks. The input stage of every op amp is a differential amplifier.
• Basic Characteristics
  – Two matched transistors with emitters shorted together and connected to a current source
  – Devices must always be in active mode
  – Amplifies the difference between the two input voltages, but there is also a common mode amplification in the non-ideal case
• Let’s first qualitatively understand how this circuit works.
  – NOTE: This qualitative analysis also applies for MOSFET differential pair circuits
Case 1

- Assume the inputs are shorted together to a common voltage, $v_{CM}$, called the common mode voltage
  - equal currents flow through $Q_1$ and $Q_2$
  - emitter voltages equal and at $v_{CM}-0.7$ in order for the devices to be in active mode
  - collector currents are equal and so collector voltages are also equal for equal load resistors
  - difference between collector voltages = 0

- What happens when we vary $v_{CM}$?
  - As long as devices are in active mode, equal currents flow through $Q_1$ and $Q_2$
  - Note: current through $Q_1$ and $Q_2$ always add up to $I$, current through the current source
  - So, collector voltages do not change and difference is still zero….
  - Differential pair circuits thus reject common mode signals

Case 2 & 3

- $Q_2$ base grounded and $Q_1$ base at +1 V
  - All current flows through $Q_1$
  - No current flows through $Q_2$
  - Emitter voltage at 0.3V and $Q_2$’s EBJ not FB
  - $v_{C1} = V_{CC} - αIR_C$
  - $v_{C2} = V_{CC}$

- $Q_2$ base grounded and $Q_1$ base at -1 V
  - All current flows through $Q_2$
  - No current flows through $Q_1$
  - Emitter voltage at -0.7V and $Q_1$’s EBJ not FB
  - $v_{C2} = V_{CC} - αIR_C$
  - $v_{C1} = V_{CC}$
Case 4

- Apply a small signal $v_i$
  - Causes a small positive $\Delta I$ to flow in $Q_1$
  - Requires small negative $\Delta I$ in $Q_2$
    - since $I_{E1} + I_{E2} = I$
  - Can be used as a linear amplifier for small signals ($\Delta I$ is a function of $v_i$)
- Differential pair responds to differences in the input voltage
  - Can entirely steer current from one side of the diff pair to the other with a relatively small voltage

- Let’s now take a quantitative look at the large-signal operation of the differential pair

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**BJT Diff Pair – Large-Signal Operation**

- First look at the emitter currents when the emitters are tied together

\[
i_{E1} = \frac{I_S}{\alpha} e^{\frac{v_{B1} - v_E}{V_T}} \quad i_{E2} = \frac{I_S}{\alpha} e^{\frac{v_{B2} - v_E}{V_T}} \quad \Rightarrow \quad \frac{i_{E1}}{i_{E2}} = e^{\frac{v_{B1} - v_{B2}}{V_T}}
\]

- Some manipulations can lead to the following equations

\[
\frac{i_{E1}}{i_{E1} + i_{E2}} = \frac{1}{1 + e^{\frac{v_{B2} - v_{B1}}{V_T}}} \quad \frac{i_{E2}}{i_{E1} + i_{E2}} = \frac{1}{1 + e^{\frac{v_{B1} - v_{B2}}{V_T}}}
\]

- and there is the constraint:

\[
i_{E1} + i_{E2} = I
\]

\[
i_{E1} = \frac{I}{1 + e^{\frac{v_{B2} - v_{B1}}{V_T}}} \quad i_{E2} = \frac{I}{1 + e^{\frac{v_{B1} - v_{B2}}{V_T}}}
\]

- Given the exponential relationship, small differences in $v_{B1,2}$ can cause all of the current to flow through one side
• Notice \( v_{B1} - v_{B2} \approx 4V_T \) enough to switch all of current from one side to the other
• For small-signal analysis, we are interested in the region we can approximate to be linear
  – small-signal condition: \( v_{B1} - v_{B2} < V_T/2 \)

**BJT Diff Pair – Small-Signal Operation**

- Look at the small-signal operation: small differential signal \( v_d \) is applied
  \[
v_{B1} - v_{B2} - v_d \Rightarrow i_{C1} = \frac{\alpha I}{1 + e^{-\frac{v_d}{2V_T}}} - \alpha I e^{\frac{v_d}{2V_T}} + e^{-\frac{v_d}{2V_T}}
  \]
- expand the exponential and keep the first two terms
  \[
i_{C1} \approx \frac{\alpha I (1 + v_d/2V_T)}{(1 + v_d/2V_T)} + \frac{\alpha I v_d}{2V_T - 2} \]
  \[
i_{C2} \approx \frac{\alpha I}{2} - \frac{\alpha I v_d}{2V_T} \]
  \[
i_c = \frac{\alpha I v_d}{2V_T} \]
  \[
g_m = \frac{I_C}{2} = \frac{\alpha I}{2V_T} \]
  \[
i_c = g_m v_d/2 \]
Differential Voltage Gain

- For small differential input signals, $v_d \ll 2V_T$, the collector currents are...

\[
\begin{align*}
    i_{C1} &= I_C + g_m \frac{v_d}{2} \\
    i_{C2} &= I_C - g_m \frac{v_d}{2} \\
    v_{C1} &= (V_{CC} - I_C R_C) - g_m R_C \frac{v_d}{2} \\
    v_{C2} &= (V_{CC} - I_C R_C) + g_m R_C \frac{v_d}{2}
\end{align*}
\]

- We can now find the differential gain to be...

\[
A_d = \frac{v_{c1} - v_{c2}}{v_d} = -g_m R_C
\]

BJT Diff Pair – Differential Half Circuit

- We can break apart the differential pair circuit into two half circuits – which then looks like two common emitter circuits driven by $+v_d/2$ and $-v_d/2$
Small-Signal Model of Diff Half Circuit

- We can then analyze the small-signal operation with the half circuit, but must remember
  - parameters $r_{\pi}$, $g_m$, and $r_o$ are biased at $1/2$
  - input signal to the differential half circuit is $v_d/2$

- voltage gain of the differential amplifier (output taken differentially) is equal to the voltage gain of the half circuit

$$A_d = \frac{v_{c1}}{v_d/2} = -g_m \left( R_C || r_o \right)$$

Common-Mode Gain

- When we drive the differential pair with a common-mode signal, $v_{CM}$, the incremental resistance of the bias current effects circuit operation and results in some gain (assumed to be 0 when $R$ was infinite)

$$v_{c1} = -v_{CM} \frac{\alpha R_C}{2R + r_e} \Leftrightarrow -v_{CM} \frac{\alpha R_C}{2R}$$

$$v_{c2} \Leftrightarrow -v_{CM} \frac{\alpha R_C}{2R}$$
Common Mode Rejection Ratio

- If the output is taken differentially, the output is zero since both sides move together. However, if taken single-endedly, the common-mode gain is finite

\[ A_{cm} = -\frac{\alpha R_C}{2R} \]

- If we look at the differential gain on one side (single-ended), we get…

\[ A_{d,\text{single-ended}} = \frac{1}{2} g_m R_C \]

- Then, the common rejection ratio (CMRR) will be

\[ \text{CMRR} = \left| \frac{A_d}{A_{cm}} \right| \cong g_m R \]

- which is often expressed in dB

\[ \text{CMRR} = 20 \log_{10} \left| \frac{A_d}{A_{cm}} \right| \]

CM and Differential Gain Equation

- Input signals to a differential pair usually consists of two components: common mode \( (v_{CM}) \) and differential \( (v_d) \)

\[ v_{cm} = \frac{v_1 + v_2}{2} \quad v_d = v_1 - v_2 \]

- Thus, the differential output signal will be in general…

\[ v_o = A_{cm} \frac{v_1 + v_2}{2} + A_d(v_1 - v_2) \]
The same basic analysis can be applied to a MOS differential pair

\[ i_{D1,2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (v_{GS1,2} - V_t)^2 \]

\[ \sqrt{i_{D1,2}} = \sqrt{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (v_{GS1,2} - V_t)} \]

- and the differential input voltage is

\[ v_{id} = v_{GS1} - v_{GS2} \]

\[ \sqrt{i_{D1}} - \sqrt{i_{D2}} = \sqrt{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} v_{id}} \]

- With some algebra...

\[ i_{D1,2} \approx \frac{I}{2} \pm \left( \frac{I}{V_{GS} - V_t} \right) \frac{v_{id}}{2} \]

\[ g_m = \frac{2I_D}{V_{GS} - V_t} = 2 \frac{I/2}{V_{GS} - V_t} = \frac{I}{V_{GS} - V_t} \]

\[ i_d = \frac{g_m v_{id}}{2} \]

We get full switching of the current when...

\[ |v_{id}|_{max} = \sqrt{2} (V_{GS} - V_t) \]
Another Way to Analyze MOS Differential Pairs

- Let’s investigate another technique for analyzing the MOS differential pair.
- For the differential pair circuit on the left (driven by two independent signals), compute the output using superposition:
  - Start with $V_{in1}$, set $V_{in2}=0$ and first solve for $X$ w.r.t. $V_{in1}$.
  - Reduces to a degenerated common-source amp.
  - Neglecting channel-length modulation and body-effect, $R_S = 1/gm_2$.
  - So...

$$\frac{v_X}{v_{in1}} = \frac{-gm_1R_D}{1 + gm_1R_S} = \frac{-R_D}{1/gm_1 + 1/gm_2}$$

- Now, solve for $Y$ w.r.t. $V_{in1}$.
- Replace circuit within box with a Thevenin equivalent:
  - $M_1$ is a source follower with $V_T = V_{in1}$.
  - $R_T = 1/gm_1$.
- The circuit reduces to a common-gate amplifier where...

$$\frac{v_Y}{v_{in1}} = \frac{R_D}{1/gm_1 + 1/gm_2}$$

- So, overall (assuming $gm_1 = gm_2$)

$$v_X - v_Y \bigg|_{\text{due to } v_{in1}} = \frac{-2R_D}{1/gm_1 + 1/gm_2}v_{in1} = -gm_R_Dv_{in1}$$

by symmetry

$$v_X - v_Y \bigg|_{\text{due to } v_{in2}} = gm_R_Dv_{in2}$$

$$A_d = \frac{v_X - v_Y}{v_{in1} - v_{in2}} = -gm_R_D$$
Differential Pair with MOS loads

- Can use load resistors or MOS devices as loads
  - Diode-connected nMOS loads = $1/g_m$ load resistance
    - Load resistance looking into the source
  - Diode-connected pMOS loads = $1/g_m$ load resistance
    - Load resistance looking into diode connected drain
  - pMOS current source loads = $r_o$ load resistance
    - Has higher gain than diode-connected loads
  - pMOS current mirror
    - Differential input and single-ended output

Consider the above two MOS loads used in place of resistors
- Left:
  - a diode connected pMOS has an effective resistance of $1/g_{mP}$
    \[ A_d = -g_{mN}(1/g_{mP})|r_{oN}|r_{oP} \approx -g_{mN} \frac{g_{mP}}{g_{mP}} \]
- Right:
  - pMOS devices in saturation have effective resistance of $r_{oP}$
    \[ A_d = -g_{mN}(r_{oN})|r_{oP} \]
Active-Loaded CMOS Differential Amplifier

- A commonly used amplifier topology in CMOS technologies
- Output is taken single-endedly for a differential input
  - with a $v_{id}/2$ at the gate of M1, $i_1$ flows
    \[ i_1 = g_m(v_{id}/2) \]
  - $i_1$ is also mirrored through the M3-M4 current mirror
  - A $-v_{id}/2$ at the gate of M2 causes $i_2$ to also flow through M2
    \[ i_2 = g_m(v_{id}/2) \]
- Given that $I_o = I/2$ (nominally)
  \[ g_m = \frac{I}{V_{GS} - V_{th}} \]
- The voltage at the output then is given by...
  \[ v_o = (i_1 + i_2)(r_{o2}||r_{o4}) = 2i_1(r_{o2}||r_{o4}) \]
  since $i_1 = i_2 = g_m(v_{id}/2)$
  \[ A_d = \frac{v_o}{v_{id}} = g_m(r_{o2}||r_{o4}) \]

Differential Amp with Linearized Gain

- Use source generation to make the gain linear with respect to the differential input and independent of $g_m$
  - Can build in two ways…
• Assuming a virtual ground at node X, we can draw the following small-signal half circuit.

- Assume $r_o$ is very large (simplifies the math)

$$v_{id} = v_{\pi} + i_s R_S$$
$$i_s = \frac{v_S}{R_S} = g_m v_{\pi} = -\frac{v_o}{R_D}$$
$$v_{\pi} = \frac{v_{id}}{1 + g_m R_S}$$

$$v_o = -g_m R_D v_{\pi}$$
$$\frac{v_o}{v_{id}} = -\frac{g_m}{1 + g_m R_S} R_D$$
$$\frac{v_o}{v_{id}} \approx \frac{R_D}{R_S} \text{ for } R_S \gg 1/g_m$$

**Offsets in MOS Differential Pair**

• There are 3 main sources of offset that affect the performance of MOS differential pair circuits
  - Mismatch in load resistors
  - Mismatch in $W/L$ of differential pair devices
  - Mismatch in $V_{th}$ of differential pair devices
• Let’s investigate each individually
Resistor Mismatch

- For the differential pair circuit shown, consider the case where
  - Load resistors are mismatched by $\Delta R_D$
    \[ R_{D1,2} = R_D \pm \frac{\Delta R_D}{2} \]
  - All other device parameters are perfectly matched
- With both inputs grounded, $I_1 = I_2 = I/2$, but $V_O$ is not zero due to differences in the voltages across the load resistors
  \[ V_O = \frac{I}{2} \Delta R_D \]
  - It is common to find the input-referred offset which is calculated as
    \[ V_{os} = \frac{V_O}{A_d} \]
  - since $A_d = g_m R_D$
    \[ g_m = \frac{I}{V_GS - V_t} \Rightarrow V_{os} = \left( \frac{V_GS - V_t}{2} \right) \left( \frac{\Delta R_D}{R_D} \right) \]

W/L Mismatch

- Now consider what happens when device sizes W/L are mismatched for the two differential pair MOS devices M1 and M2
  \[ \left( \frac{W}{L} \right)_{1,2} = \frac{W}{L} \pm \frac{1}{2} \Delta \left( \frac{W}{L} \right) \]
  - This mismatch causes mismatch in the currents that flow through M1 and M2
    \[ I_{1,2} = \frac{I}{2} \mp \frac{I}{2} \left( \frac{\Delta (W/L)}{2(W/L)} \right) \]
  - This mismatch results in $V_O$
    \[ V_O = IR_D \frac{\Delta (W/L)}{2(W/L)} \]
  - So in the input referred offset is...
    \[ V_{os} = \frac{V_O}{A_d} \Rightarrow V_{os} = \left( \frac{V_GS - V_t}{2} \right) \left( \frac{\Delta (W/L)}{(W/L)} \right) \]
**V_t Mismatch**

- Lastly, consider mismatches in the threshold voltage
  \[ V_{t1,2} = V_t \pm \frac{\Delta V_t}{2} \]

- Again, currents \( I_1 \) and \( I_2 \) will differ according to the following saturation current equation
  \[ I_{D_{sat1}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left( V_{GS} - V_t - \frac{\Delta V_t}{2} \right)^2 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2 \left[ 1 - \frac{2(\Delta V_t)}{2(V_{GS} - V_t)} \right]^2 \]
  - For small \( \Delta V_t << 2(V_{GS} - V_{th}) \)
  \[ I_{D_{sat1,2}} \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2 \left( 1 + \frac{\Delta V_t}{V_{GS} - V_t} \right) = \frac{I}{2} \pm \frac{\Delta I}{2} \text{ where } \Delta I = \frac{I}{2} \left( \frac{\Delta V_t}{V_{GS} - V_t} \right) \]
  - Again, using \( V_{OS} = V_O/A_d \) (\( A_d = g_m R_D \) and \( V_O = 2\Delta I R_D \)) we get...
  \[ V_{os} = \frac{2(\Delta I)(R_D)}{A_d} = \frac{2IR_D}{2} \left( \frac{\Delta V_t}{V_{GS} - V_t} \right) \frac{V_{GS} - V_t}{IR_D} = \Delta V_t \]

**Mismatch Summary**

- The 3 sources of mismatch can be combined into one equation:
  \[ V_{os} = |\Delta V_t| + \left| \frac{V_{GS} - V_t}{2} \left[ \frac{\Delta R_D}{2 R_D} + \frac{\Delta (W/L)}{(W/L)} \right] \right| \]
  - arising from \( V_t, R_D, \) and \( W/L \) mismatches
- Notice that offsets due to \( \Delta R_D \) and \( \Delta W/L \) are functions of the overdrive voltage \( V_{GS} - V_t \)